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DETERMINING WHETHER THE CONTENTS OF A DATA FILE ARE SUFFICIENT TO FILL A RETRIEVAL REQUEST

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This paper is a direct extension of ideas developed in my S.M. Thesis. I am indebted to J. C. Emery and D. C. Carroll for their aid.

We present a solution to a problem of considerable practical interest which arises in the design of Management Information Systems. After presenting an example of the problem, we shall formalize it.

Example: Suppose we have a personnel file containing complete data on all our 20,000 workers. The Operations Research Group, wishing to conduct a study of absenteeism among older workers, has created a much smaller file containing the records of all those aged 60 or over as well as those of any age who were absent 10 or more days in the last year. Let us suppose that this file is 1500 records long.

The personnel department now wishes to obtain the names of all employees within 2 years of compulsory retirement at age 65. The desired list can be obtained either from the large file owned by the personnel department or from the smaller file owned by the Operations Research Group. Question: How would the information system "know" that the list could be obtained from the shorter file (at some considerable saving)?

Formalization: Let a "record" be a collection of integers $k_1,\ldots,k_j,\ldots,k_u$. In general, the value of k_j for any record implies nothing whatsoever about the value of k_j for another record. Let a "file" be a collection of m records. Let there be a special file called the "master file" which consists of all records of interest.

Let a "request" R_i be a function which maps the integer contents of a record into the values "true" and "false". Let a "strip file" S_i be a file produced as follows: The master file is read record by record. As each

record is read, the request R_i is applied to the integer contents of that record. If and only if the value of R_i is "true", the record is copied to the strip file S_i.

Further, let the data base at any moment consist of the master file and p strip files $S_1,\ldots,S_i,\ldots,S_p$, produced from requests $R_i,\ldots,R_i,\ldots,R_p$, respectively.

Now, a new request R_{p+1} is entered into the system. If another request, say R_a, can be found such that R_{p+1} implies R_a, then it must be true that all of the records in the master file which would cause R_{p+1} to be true are also in S_a. In other words, the records in S_{p+1} will be a subset of the records in S_a. In this case, there is probably an advantage in creating S_{p+1} from S_a rather than from the master file.

Procedure: Determining whether some request R_c implies some other request R_d can be accomplished by a technique called "recursive residual reduction".[1] First let us consider requests formed in terms of logical variables (i.e., those taking on only the values "true" and "false" (abbreviated as 1 and 0 respectively); three Boolean operators * ("and"), + ("inclusive or"), and ' ("not"); and parentheses; all with their customary meanings. (We assume that * takes precedence over + in the hierarchy of operators.)

We will use the capital letters A-H as logical variables. Consider two requests:

$$R_1 = A+B$$

$$R_2 = A*(C+B)'$$

R_1 is true if either A or B (or both) is true, and R_2 is true if A is true, but neither B nor C is true. R_2 implies R_1, since R_1 is always true if R_2 is true. This fact can be determined by a simple procedure, based on the "Expansion Theorem"[2], which states that $f(x_1,\ldots,x_i,\ldots,x_n) = x_i * f(x_1,\ldots,1,\ldots,x_n) + x_i' * f(x_1,\ldots,0,\ldots,x_n)$ where f is any Boolean expression in logical variables $x_1,\ldots,1,\ldots,x_n$. The expressions $f(x_1,\ldots,1,\ldots,x_n)$ and $f(x_1,\ldots,0,\ldots,x_n)$ are called the "residue about x_i" and the "residue about x_i'" respectively. Implication can be determined by the following recursive procedure:

Let U and V be two Boolean expressions.

1. U does not imply V if U = 1 and V = 0.

2. U implies V is U = 0 or V = 1 or both.

3. If neither rule 1 nor rule 2 applies, choose a logical variable, say Q, appearing in either U, V, or both. (Call this variable the "pivot variable".) Take residues of U and V about Q and Q'. U implies V if and only if the residue of U about Q implies the residue of V about Q and the residue of U about Q' implies the residue of V about Q'.

Reducing a residue to simpler form is achieved by applying the following rules: Let E be any logical expression (i.e., a logical variable, a constant 0 or 1, or an expression enclosed in parentheses.

a) 1 + E = E + 1 = 1

b) 1 * E = E * 1 = E

c) 0 + E = E = 0 = E

d) 0 * E = E * 0 = 0

e) 0' = 1

f) 1' = 0

g) parentheses should be removed where not needed for operations hierarchy, i.e., from any single logical variable or constant; or from a complete expression where neither preceded nor followed by an operator.

Let us go back to $R_1 = A + B$

$$R_2 = A*(C+B)'$$

and apply our rules to see whether R_2 implies R_1.

First, we see that neither rule 1 nor rule 2 applies directly, so invoking rule 3 and choosing A as pivot variable, we have

	Residue about A	Residue about A'
R_1	$1 + B = 1$	$0 + B = B$
R_2	$1 *(C+B)' = (C+B)'$	$0 *(C+B)' = 0$

By rule 2, it will be seen that the residue of R_2 about A implies the residue of R_1 about A (because the residue of R_1 about A is 1); and the residue of R_2 about A' implies the residue of R_1 about A' (because the residue of R_2 about A' is 0). Therefore R_2 implies R_1.

In practice we would probably not represent the logical expressions symbolically, but rather as a "Polish" string of operators and operands with all negations resolved by De Morgan's theorem.[3]

Now let us return to our original problem of two personnel files. We must extend our language to include the relational operators = , $\neq$, $>$, $\geq$, $<$, $\leq$, . Suppose that integers k_i and k_j represent age as of January 1 and days absent last year respectively. The file owned by the Operations Research Group was produced by the request $R_3 = (k_i \geq 60) + (k_j \geq 10)$. The file desired

by the personnel department could be described by $R_4 = (k_i \geq 63)$. Our first step is to create logical variables corresponding to each relational operation. Let:

$$A = (k_i \geq 60)$$
$$B = (k_j \geq 10)$$
$$C = (k_i \geq 63).$$

That we are permitted to do this is clear, since a relational operation is, for any record, either true or false. Our requests are now

$$R_3 = A + B$$
$$R_4 = C$$

But we know that both A and C are derived from the same integer, namely, k_i. In fact, if C is true, A must also be true. Therefore, C implies A. Recognizing this fact makes us treat the Expansion Theorem more generally: Any residue about C will substitute 1 for <u>both</u> A and C. Any residue about A' will substitute 0 for <u>both</u> A and C.

The reasoning behind this is clear. The residue of U about X is simply the form of U, remaining if X is <u>assumed</u> true. But if X implies Y, then the residue of U about X must also assume Y to be true. Similarly, the residue of U about Y' must assume X to be false.

Let us now try the generalized procedure on R_3 and R_4, remembering that C implies A.

R_3 A + B

R_4 C

Neither rule 1 nor rule 2 applies so applying rule 3 and choosing B as pivot variable, we have

	Residue about B	Residue about B'
R_3	A + 1 = 1	A + 0 = A
R_4	C	C

The residue of R_4 about B implies the residue of R_3 about B by rule 2, but neither rule 1 nor rule 2 applies to the residues about B', so we invoke rule 3, choosing C as pivot variable:

	Residue about C	Residue about C'
Residue of R_3 about B'	A = 1 (by C implies A)	A
Residue of R_4 about B'	1	0

Both residues of R_4 imply their corresponding residues of R_3, therefore R_4 implies R_3.

Extensions: The general procedure descirbed herein is capable of a great many extensions:

1. File structure -- Our definition of a master file is phrased so that it includes files stored on direct-access devices as well as sequential devices. The notion of a strip file is easily generalized then to a list of those records stored in a direct-access master file which meet some criterion (request).

2. Other Relationships -- Equivalence, negation, contraimplication, etc., can be determined by simple modifications of the procedure described here.[4]

3. Other Operations -- All logical functions of two variables (there are at most 16) can be easily handled by appropriate additions to rules a) - g). Moreover, one practical situation which we have recently faced required the definition of n-ary equivalents of "or" and "and".[5] Extension of this technique to n-ary operations is straight forward.

4. Finding the shortest qualifying strip file -- If we order strip files $S_1 \ldots S_p$ in ascending order of length, then iteratively apply this technique to each R_i in turn with respect to R_{p+1}, the first R_i implied by R_{p+1} is the shortest qualifying file. If none qualify, the master file must be used.

Conclusion

The "Expansion Theorem" is a powerful tool in the analysis of the contents of data files.

REFERENCES

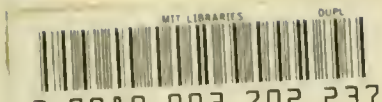
1. See Sprague, C.R., "On Efficient Multiple-User Date Retrieval From Serial Files", unpublished S.M. Thesis, M.I.T., 1964.

2. S.H. Caldwell, Switching Circuits and Logical Design, Wiley, New York, 1958, Chapter III, Theorem 20.

3. Sprague, op. cit.

4. Sprague, op. cit., Chapter IV. Section B.

5. A paper is forthcoming.



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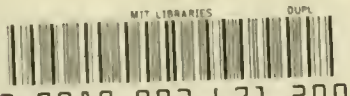

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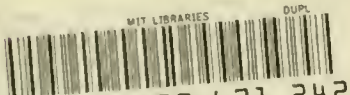

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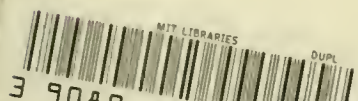

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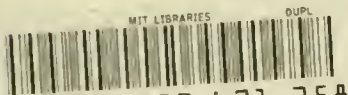

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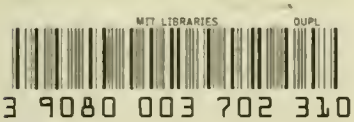

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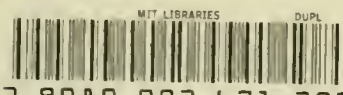

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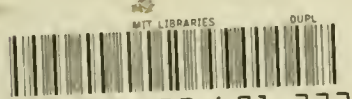

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